MATRIX ELEMENTS WITH WILSON FERMIONS

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Highlights of the results for the spectrum, meson decay constants f_π and f_V^{-1}, the chiral parameters m_q and $\bar{\psi}\psi$, and the Kaon B Parameter are presented. The calculation was done using 35 quenched $16^3 \times 40$ lattices at $\beta = 6.0$ using Wuppertal and Wall smeared sources. We show that smeared sources improve the signal significantly, consequently we are able to improve the quality of results for a number of the phenomenologically interesting quantities.

1. INTRODUCTION

In this talk I summarize results obtained with Wilson fermions using smeared sources. The details are given in Refs. [1] and [2], and I use the same notation to briefly describe the new developments. The calculation was done using two values of the quark mass corresponding to a pion mass of ≈ 660 and 540 MeV. Periodic boundary conditions were used in all four directions in calculating the quark propagators. In the analysis we only consider hadrons made of degenerate quarks, so our results have maximum validity in the limit of SU(3) flavor symmetry.

2. SURPRISES IN HADRON SPECTRUM

The two motivations for using non-local sources are (a) to improve the overlap with the desired lowest state and (b) to improve the signal in the long-time behavior of the 2-point correlators. The results of our comparative study are that this improvement is achieved both with the Wuppertal source (we used a smearing radius of
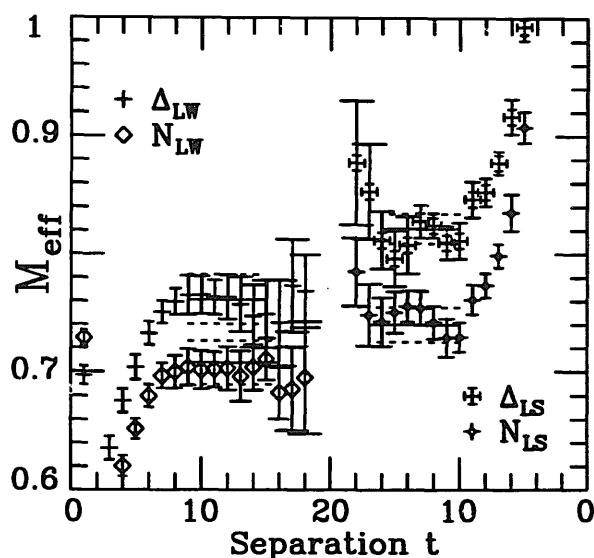

Fig. 1. Comparison of the effective mass for the nucleon and Δ obtained using Wuppertal (LS) and wall source propagators (LW).

4-4.5 lattice units) and with wall source propagators. There are clear plateaus in the effective mass plots for the π and ρ channels and one can extract the asymptotic mass estimate with confidence. The plateaus in the baryon channels (Nu-

cleon and Δ) extend over 6-8 time-slices with each of the two sources and one gets what looks like a convincing signal, however, comparing the results from the two sources exposes a disturbing feature. The two estimates are significantly different; the wall source estimate are $1-3$ standard deviations lower as shown in Fig. 1. It is not yet known whether this difference is present because neither estimate is asymptotic or due to a source dependent finite volume effect. Therefore, we advocate further tests using different sources, operators and boundary conditions to determine how best to extract asymptotic mass estimates for baryons.

Fig. 2. Comparison of lattice results for the decay constant f_V^{-1} with phenomenological estimates.

3. MESON DECAY CONSTANTS

To calculate decay constants using quark propagators with non-local sources one needs to calculate both smeared-local and smeared-smeared correlators. The relevant matrix element can then be extracted from the amplitude of the two 2-point correlator. As shown in Ref. [1] (see Eq. 5.2), one can combine the two correlators in a number of related ways, and the difference between the results is a measure of the systematic errors. We used four such combinations to determine f_π and two for f_V, and find that the results are consistent — the variation between methods is comparable to the statistical errors. Furthermore, at Lattice91 new results from the three groups APE [Ref. 5], QCDPAX [Ref. 6], and us were found to be in good agreement. A major improvement over previous calculations is seen for f_V; the new results are in much better agreement with experimental data as shown in Fig. 2.
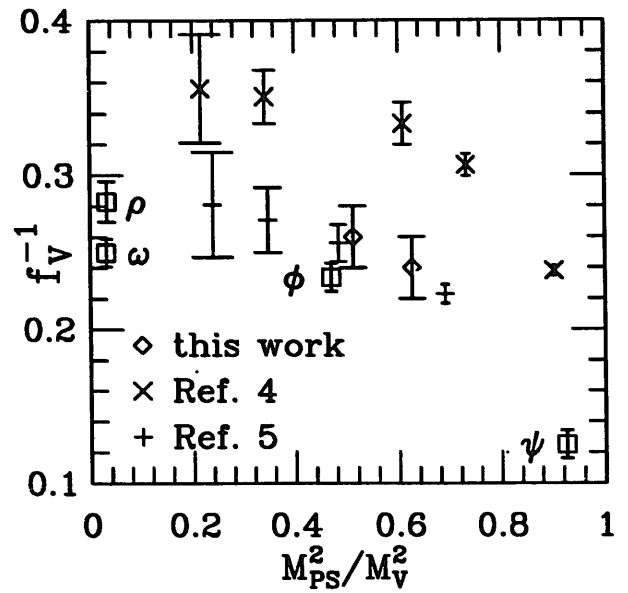
4. CHIRAL PARAMETERS m_q and $\bar{\psi}\psi$

With Wilson fermions the chiral parameters m_q and $\langle\bar{\psi}\psi\rangle_{m_q=0}$ have to be extracted from combinations of 2-point correlators. In Ref. 1 (see Eqs. 7.4-7.10) we describe the different combinations of correlators that can be used and show they give consistent results. To get $\langle\bar{\psi}\psi\rangle_{m_q=0}$, we show that a linear extrapolation to the chiral limit of results obtained at finite m_q is reliable. To compare lattice results with continuum values one has to include the renormalization constants Z_A, Z_P and Z_S, which are not well determined (estimates made using perturbation theory can be uncertain by a factor of up to 2). Present lattice estimates are not in agreement with phenomenological values; we find that m_s comes out a factor of 2-3 too small while $\langle\bar{\psi}\psi\rangle_{m_q=0}$ is correspondingly a factor of 2-3 too large. Including the Z factors significantly reduces the difference, however, we cannot yet ascertain how much of

the remaining discrepancy is due to quenching. In any case we need to determine the renormalization constants much more precisely, or better still use an improved action for which the Zs take on values closer to 1.0 and have much smaller $O(a)$ artifacts.

In previous calculations one has found a large difference between estimates obtained using Wilson and staggered fermions. The good news is that on comparing results for Wilson fermions with those from staggered fermions [Ref. 3] we find consistency once one takes into account the sizable systematic and statistical errors in the two estimates. For this comparison we used the same set of lattices at $\beta = 6.0$ and used smeared propagators in both cases.

5. KAON B PARAMETER

In order to extract B_K (for background and phenomenological implications see talk by Martinelli, Ref. 7) we wish to calculate the matrix element (ME)

$$\mathcal{M}_K(p) = \langle K(p=0)|(\bar{s}\gamma_\mu Ld)(\bar{s}\gamma_\mu Ld)|\overline{K}(p)\rangle .(1)$$

In chiral perturbation theory it behaves as $\mathcal{M}_K(p) \sim \gamma_K p_K \cdot p_{\overline{K}}$ where $\gamma_K = 8/3 \, f_K^2 B_K$, and p_K and $p_{\overline{K}}$ are the on-shell four-momenta of the external states, so that $p_K \cdot p_{\overline{K}} = M_K \sqrt{M_K^2 + (p)^2}$. Unfortunately, with Wilson fermions chiral symmetry is broken explicitly by the r term and the expansion takes the form

$$\mathcal{M}_K(p) = \alpha + \beta m_K^2 + (\gamma + \gamma_K)p_K \cdot p_{\overline{K}} + \cdots ,(2)$$

where the terms proportional to α, β and γ are unphysical contributions and suppressed by one power of the lattice spacing a. Using the perturbatively improved operator $\hat{O}$ (see Eq. 2.3 in Ref. 2) should reduce the lattice artifacts, but will not eliminate them completely because it is

only an approximation to the operator with the desired continuum behavior. One can eliminate both α and β at a fixed value of M_K by using the non-perturbative method of momentum subtraction [Refs. 2, 8]. For example, by calculating the matrix element of $\hat{O}$ for two different values of p and taking the difference one gets

$$\mathcal{M}_K(p) - \mathcal{M}_K(0)$$
$$= (\gamma + \gamma_K)M_K(E(p) - M_K) + \cdots . \quad (3)$$

In practice we calculate

$$B_K = \frac{E(p) \, B_K(p) - M_K \, B_K(0)}{E(p) - M_K} \quad (4)$$

at each value of κ, where by $B_K(p)$ we mean the ratio of the matrix element to its VSA value, both calculated on the lattice at finite momentum transfer. For this method to be viable there should be a signal in the correlators at finite momentum, and p should be small such that the quartic and higher terms neglected in Eq. 2 are small. This method does not remove the third unphysical coefficient γ; we hope that, in a future calculation, using $\hat{O}$ with an improved action will take care of it.

The calculation of the matrix elements using smeared sources is done as follows: a wall source at $t = 0$ produces a zero momentum kaon which propagate for a time t, at which point the operator inserts momentum p, and the resulting $\overline{K}$ with momentum p then propagates the remaining $(N_t\text{-}t)$ timeslices until it is destroyed by a Wuppertal source. As a consistency check we show, in Ref. [1], that the Wuppertal source correlators have significant overlap with the lowest few momenta allowed on the lattice, and the zero momentum wall source pion "reaches" the region in which there is a signal for the non-zero momentum kaon. An example of the quality of the signal for B_K using $\hat{O}$ is shown in Fig. 3.
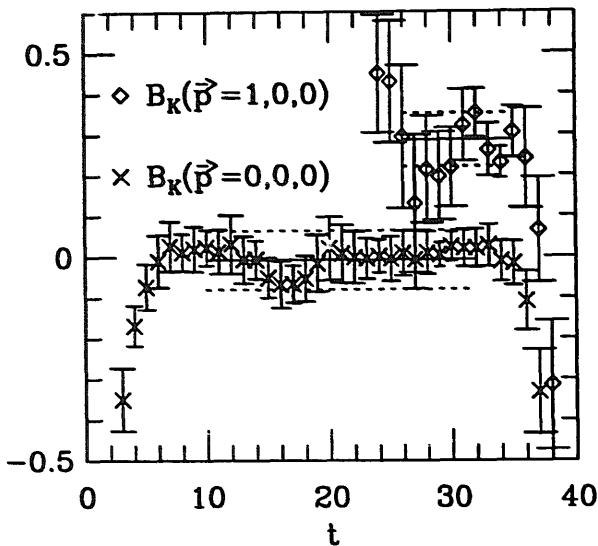
Fig. 3. The expected signal for B_K is a constant. For p = (1,0,0) there is a signal only for $t > 22$ where the effective mass plot shows a plateau.

In order to study the effect of the mixing due to the r term we have analyzed B_O separately for each of the 16 gamma matrix structures and the two possible contractions. We find that there is a large cancellation of the different ME, consequently since the coefficient of the perturbative correction is $\approx 2. \times 10^{-3}$ the mixing induces at most a $2 - 3\%$ effect. Similarly, the 1-loop renormalization of the 4-fermi LL operator is largely cancelled by Z_A^2 needed to renormalize the VSA. Thus, overall the 1-loop corrections are $5 - 10\%$, i.e. comparable to the statistical errors in individual ME.

Our best estimates after performing the momentum subtraction are

$$B_K(\kappa = 0.154) = 0.69(25) ,$$

$$B_K(\kappa = 0.155) = 0.74(27) . \qquad (5)$$

Our data give $\alpha \sim 0.003$, $\beta \sim 0.02$ and $\gamma + \gamma_K \sim 0.015$, which shows that the lattice artifacts are large.

We compare our Wilson results with Staggered data at same β and roughly the same pseudoscalar masses as given in Ref. [9]. The estimates of B_K are consistent, though the errors in the Wilson estimate are larger by a factor of 10, a large part of which is due to the process of momentum subtraction. In addition, we mention that the individual axial-axial and vector-vector terms are also comparable and furthermore, show similar chiral behavior [Ref. 2].

To conclude, we believe that the method of momentum subtraction works and future high statistics calculations using an improved action will allow us to calculate B_K reliably with Wilson fermions also.

ACKNOWLEDGEMENTS

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